Wales HIGH SCHOOL SIXTH FORM

Academic Reading List

Mathematics

Mathematics

Hello new Y12 Mathematics Students!

The transition from GCSE Maths to A level Maths can be quite daunting but we are here to support you.

Below are probably the two most important topics to make sure you are fully conversant with before September.

Having worked with some of you on Taster Days I know you will be amazing.

If you need any further support with this there are many videos on Youtube you can look at. For instance exam solutions – the link is below. There are many others!

https://www.youtube.com/user/examsolutions

Our students are becoming very adept at looking through Physics and Maths Tutor for extra support too. It is amazing!

https://www.physicsandmathstutor.com/

Of course you can email me on cbb@waleshigh.com if you need any support from me or have any questions.

Good Luck and please know we are here to support you from day one!

We look forward to working with you,

Mr Bonnen-Brailsford and the Maths team.

Mathematics

Key Skill 1: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

Method 1: Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example 1: Solve $x^2 - 3x + 2 = 0$

Factorise $(x-1)(x-2) = 0$
Either $(x-1) = 0$ or $(x-2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Example 2: Solve $x^2 - 2x = 0$

Factorise: $x(x-2) = 0$
Either $x = 0$ or $(x-2) = 0$
So $x = 0$ or $x = 2$

Method 2: Using the formula

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Solve the equation $2x^2 - 5 = 7 - 3x$

Solution: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

If we have a calculator, we can evaluate these roots to get: $x = 1.81$ or $x = -3.31$

If you need more help with the work in this chapter, there is an information booklet downloadable from this web site:

Mathematics

EXERCISE

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations to 3 significant figures. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

Mathematics

Key Skill 2: INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of y.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$
3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the a's)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the c's)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the d terms i.e. by subtracting the powers)

More complex powers

Zero index:
Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number a.

Therefore $5^0 = 1$ $\left(\dfrac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \dfrac{1}{a}$

Therefore $5^{-1} = \dfrac{1}{5}$

$0.25^{-1} = \dfrac{1}{0.25} = 4$

$\left(\dfrac{4}{5}\right)^{-1} = \dfrac{5}{4}$ (you find the reciprocal of a fraction by swapping the top and bottom over)

This result can be extended to more general negative powers: $a^{-n} = \dfrac{1}{a^n}$.

This means:

$3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

Fractional powers:

Fractional powers correspond to roots:

$$a^{1/2} = \sqrt{a} \qquad a^{1/3} = \sqrt[3]{a} \qquad a^{1/4} = \sqrt[4]{a}$$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$$8^{1/3} = \sqrt[3]{8} = 2 \qquad 25^{1/2} = \sqrt{25} = 5 \qquad 10000^{1/4} = \sqrt[4]{10000} = 10$$

A more general fractional power can be dealt with in the following way: $a^{m/n} = \left(a^{1/n}\right)^m$

So $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{2/3} = \left(\left(\frac{8}{27}\right)^{1/3}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-3/2} = \left(\frac{36}{25}\right)^{3/2} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) 5^{-2}

5) 18^0

6) 7^{-1}

7) $\left(\frac{2}{3}\right)^{-2}$

8) $(0.04)^{1/2}$

9) $\left(\frac{8}{27}\right)^{2/3}$

Simplify 10) $2a^{1/2} \times 3a^{5/2}$ 11) $x^3 \times x^{-2}$ 12) $\left(x^2 y^4\right)^{1/2}$